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Section 1-2 Number systems and conversions
  Thursday, January 7, 2021
              4:23 PM
  Computers do everything in binary (base 2), so we have to be proficient at
  converting from one base to another
  Positional Notation
  Consider the number 253.82 in base 10 (253.82<sub>10</sub>)
  Each digit is multiplied by a power of 10:
  253.82=
  In bese 2 a number would be:
   1100.01, =
  The base is called the radix R
    Eq. base 8 has radix R=8
  Digits go from O to R-1
    Ey for R = 8, digits are 0,1,2,3,4,5,6,7
 A number represented by
   N= (... ay 9, a, a, a, a, a, ...)
 is ... a, xR' + a, x R3 + a, x R2 + a, x R' + a, x R2+
         + a-1×R-1 + a-2×R-2 -.
Example:
      7,659.20
   a4=0 a3=7 a2=6 a,=5 a0-9 a1=2 a=0
7,659.2,0 = 0x104 + 7x103 + 6x102 + 5x101 + 9x100 + 1x101
 -O+ 7000 + 6 00 +
Called a power series expansion
Example: write 147, as a power series expansion
   147,0=
Suppose we wanted to write 1970 in base 3
   10=13 40=113 7,0=213 and 10,0=1013
 147, = 13 × (103) + 113 × (1013) + 213 × (1013)
  technically correct, but you'd have to
         do the arithmetic in base 3 -
 Protrp: Use power series expansion when converting
 from some base TO base 10
 Example 2415 -> bese 10?
      start at do (just to left of decimal point)
       a_0=1 a_1=4 a_2=2
   ·· N = 2x52 + 4x51 +1x5°
        = 50 + 20 + 1
What if R > 10?
       need 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, ????
 Hexadecimal (Base 16)
   A = 10, B= 11, C=
Example
    F5016 = ? R=16
     a = C = 12,0
     a, = 5,6 =
      a2= F16 =
F5C16 = F x (1610) +
| Pro tip: write the base on every number to be clear
 Exponents are always understood to be base 10
     so very confusing! e.g. A^2 = A \times A
    if for some reason we wanted B wild write
Converting FROM base 10
    (will deal with integers first, come back to
       decimals)
   Continuous division method:
  lour number in base R is
       N= (a, a, a, a, ... a, a,),
N = \alpha_n R^n + \alpha_{n-1} R^{n-1} + \cdots + \alpha_i R^i + \alpha_o R^i
  Now diride by R

\frac{N}{R} = a_n R^{n-1} + a_{n-1} R^{n-2} + \dots \cdot a_n R^{n-1} + a_n

      = Q, with remainder ao
  Divide again:
   \frac{Q_1}{R} = a_n R + a_{n-1} R + \dots + a_2 R^0 + a_n
       = Q2 with remainder a,
  Vivide again
   \frac{Q_2}{R} = a_n R + a_{n-1} R + \cdots + a_3 R^0 + a_2
       = Q3 with remainder a2
  keep going until you run out of numbers
   This gives is all the a's (a, a, a, a, ... etc)
 Then NR = (anan-1 an-2 --- a, a, )R
  Example
    Convert 87,0 to have 4
  Ny = (an an-1 an-2 ... a, a) y & this is what
      4/87 = 21 with remainder 3=00
      4/21 = 5 w.h remainder 1=a,
                 with remainder 1=az
      4/5 =
      45
   -: 87<sub>10</sub> =
   Remember: first remainder 15 LEAST significant disit
  Dummary so for
     · To convert to base 10, use power series
           expansson
      · To convert Fram base 10, use successive division
           (mteger)
   Converting decimals From base 10
    Consider a décimal fraction (e.g. 0.123)
         We want to convert FROM base 10 TD base R
         Fo = (a, a, a, a, ... a, m) R
            = \alpha_{-1} R^{-1} + \alpha_{-2} R^{-2} + \alpha_{-3} R^{-3} + \cdots + \alpha_{-m} R^{-1}
    Multiply by R
      For = a -1 + a -2 r + a -3 r + ···· a -m r
           = a-1+
   Multiply by Ragain
                                                     -(m-2)
     F, R = a-2 + a-3 R + ay R + --- a - m R
       keep going onthe you're done
  Example
                                        (R=2)
     Convert 0.5625,0 to base 2
     Fo = 0.56 25
    2 × Fo = 1.125 => : a = = + F = 0.125
    2xF,=0,25 -> a-2=0
                                    F_{2} = 0.25
                                     F_3 = 0.5
   27F_2 = 0.5 \rightarrow 0.5 = 0
                                   Fy = 0 done.
   2xF_3 = 1.0 \rightarrow \alpha_{-4} = 1
    This one terminated nicely.
 When it doesn't terminate, you'll get a repeating
   traction, such as
          0.7, = 0.10110 v110 2
  Example Convert 57.05, to base 3 (R=3)
   Integer part: (successive divison)
          19 remainder a_0 = 0
                                        3+19=57!
                                          For Pele's salec'
  Decimal part: For 0,05
         Sucessive multiplication
      3 \times F_{o} = 0.15 (a_{-1} = 0, f_{1} = 0.15)
      3 × F1 =
   Converting from binary to hex and back
       Dead casy! Because every 4 bits in binary
         represents l'hex dry.t
    Example
             100111.0101112 =?
            100111.010111
                                          2 = [?]
             What is A1.3, in base 2?
     Simmary (we'll settle by poll)
        To convert from some base TO
             buse lo, use
       To convert a decimal integer Fram
            base 10, use
       10 convert a décimal décimal Fram
           base 10, use
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