Section 1-3 Binary Arithmetic

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Computers work fundamentally in binary
Computers do math
To design a computer to do math, you have to know how to do it yourself.

Addition

Rules

$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 0$ and carry the "1" to the next column

Example $12_{10} + 11_{10}$ → do it in base 2

first convert base 10 numbers to base 2

2 | 12   2 | 11

now add

$12_{10} = 1100_2$
$11_{10} = 1011_2$

now check result:

$1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
$16 + 0 + 4 + 2 + 1 = 23$ ✓

Example $5_{10} + 5_{10} + 5_{10} = ?$ $5_{10} = 101_2$

```
  101
  101
  101
-----
```

check: $1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = ?$

Example $7_{10} + 7_{10} + 7_{10} = ?$ $7_{10} = 111_2$

```
  111
  111
  111
-----
```

first colum: 1+1+1 = 11
carry the "1" to second column

second colum: 1+1+1+1 = 100
this will get confusing trying to carry "10"

PRO TIP: do one sum at a time

```
7₁₀     111
7₁₀     111
---    ----
14₁₀
        111
+7₁₀   ----
----
21₁₀
```

check result:

Subtraction

Rules

$0 - 0 = 0$
$0 - 1 = 1$ → and borrow from the next column
$1 - 0 = 1$
$1 - 1 = 0$

Recall in Base 10:

```
 342
 173
----
```

In base 8

```
 342
 173
----
```

In base 2, we'll borrow "2" like this

```
  10      =2
-  1      -1
---      ---
   1       1
```

Example

```
  10101   → 16+4+1 = 21
- 10011   → 16+2+1 = 19
-------
```

check:

Example

```
  1000   → 8
-    1   → 1
------
```

Example

```
  111001
-    111
--------
```

Multiplication

Rules

$0 \cdot 0 = 0$
$0 \cdot 1 = 0$
$1 \cdot 0 = 0$
$1 \cdot 1 = 1$

Example (keep your columns straight!)

```
   1001   → 8+1 = 9₁₀
x  1101   → 8+4+1 = 13₁₀
-------     should get 117₁₀
```

check 1110101

$1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
$64 + 32 + 16 + 0 + 4 + 0 + 1$
$= 117$ ✓

Example

```
   1011
x  1111
-------


-------
```

PRO TIP: do sums one at a time

```
    1011
x   1111
--------
   11011     first product
+ 1011       second product
--------     watch colums!
  100001     third product
```

Division

Inside-out multiplication – but much easier!

Look:

```
1011 ) 10010001
```

- 1011 does not go into 1
- 1011 does not go into 10
- 1011 does not go into 100
- 1011 does not go into 1001

Example

```
11 ) 100
```

- 11 does not go into 1
- 11 does not go into 10
- 11 Does go into 100 and it has to be 1 time

Example

```
101 ) 010000
```

Examp

```
1011 ) 10010001
```

Next up: negative binary numbers