Section 2-5 Commutative, Associative, Distributive, and De Morgan's Laws

Wednesday, January 27, 2021 11:03 AM

Other useful Laws (some new and exciting!)

Commutative

$XY = YX$ $X + Y = Y + X$

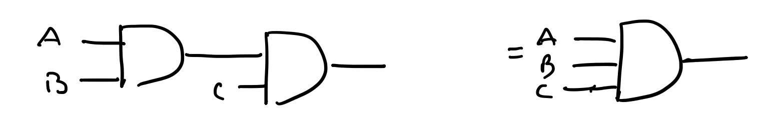
Associative

$(XY)Z = X(YZ) = XYZ$ $(X+Y)+Z = X+(Y+Z) = X+Y+Z$

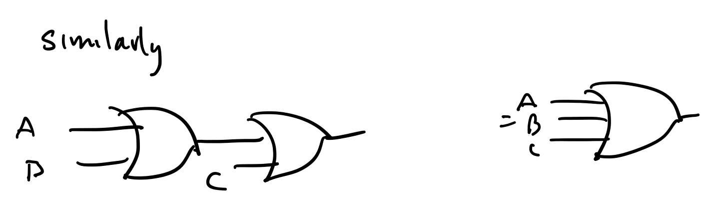
Proof by truth table: prove $(XY)Z = X(YZ)$

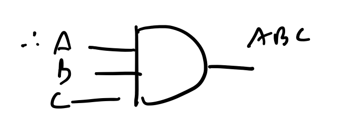
X	Y	Z	XY	YZ	(XY)Z	X(YZ)
0	0	0				
0	0	1				
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

In gate land

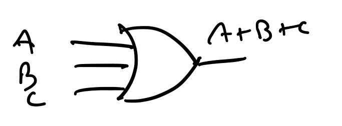


similarly





output is "1" iff every input is 1



output is "0" iff every input = 0

Distributive Law

① $X(Y+Z) = XY + XZ$

But wait! In Boolean algebra, there is a SECOND distributive law that doesn't work in regular algebra:

② $X + YZ = (X+Y)(X+Z)$

> Compare:
>
> ① $X \cdot (Y+Z) = X \cdot Y + X \cdot Z$
>
> ② $X + Y \cdot Z = (X+Y) \cdot (X+Z)$
>
> swapped AND's and OR's

Proof of 2nd Distributive Law

$(X+Y)(X+Z) = X(X+Z) + Y(X+Z)$ by 1st distributive

Put another way: in Boolean algebra,
AND distributes over OR (1st dist)
OR distributes over AND (2nd dist)

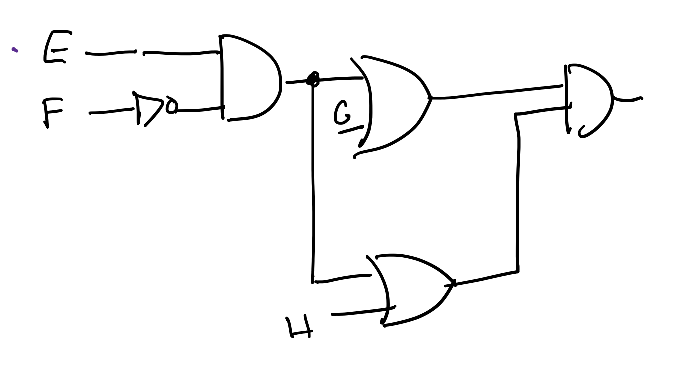
$A + BC$ cannot be further factored in regular algebra

$A + BC = (A+B)(A+C)$ in Boolean algebra

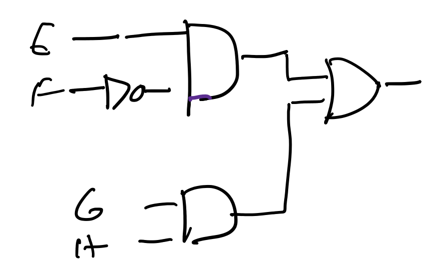
Example simplify $(EF' + G)(H + EF')$
notice "EF'" is common to both

let $A = EF'$, $B = G$, $C = H$
can write expression as $(A+B)(A+C)$ (commutative)
$= A + BC$ 2nd dist
$= EF' + GH$

How many gates does original have?



and simplified?



De Morgan's Laws

$(X+Y)' = X'Y'$
$(XY)' = X' + Y'$

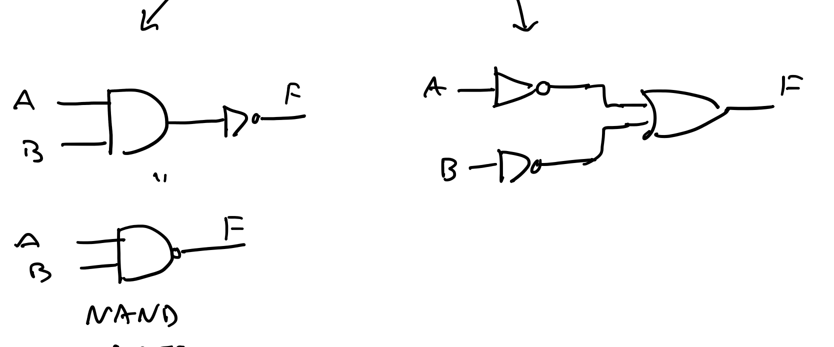
Based on "dualities"
"AND" is the dual of "OR"
A' is the dual of A
1 is the dual of 0

Suppose $F = (X + X')$ ① go from left to right take the dual of everything
$= (X' \cdot X) = 0$

Example
$Z = (A'BC + A'BC')'$

think of as $(A' \cdot B \cdot C \oplus A' \cdot B \cdot C')'$

Example $F = (AB)' = A' + B'$
NAND   OR



Verify De Morgan's Laws via truth table
$(X+Y)' = X'Y'$
$(XY)' = X' + Y'$

X	Y	X'	Y'	X+Y	(X+Y)'	X'Y'	XY	(XY)'	X'+Y'
0	0								
0	1								
1	0								
1	1								

(X+Y)' and X'Y': should be same
(XY)' and X'+Y': should be same

Summary so far (p. 46 in book)

Operations with 0 and 1

$X + 0 = X$ duals $X \cdot 1 = X$
$X + 1 = 1$ $X \cdot 0 = 0$

Idempotent

$X + X = X$

Involution

$(X')' = X$ (no dual)

Law of complementarity

$X + X' = 1$

Commutative

$X + Y = Y + X$

Associative

$(X+Y)+Z = X+(Y+Z)$

Distributive

$X(Y+Z) = XY + XZ$

De Morgan's

$(X+Y)' = X'Y'$