Section 2-6 Simplification theorems

Thursday, January 28, 2021 5:37 PM

You can never have enough theorems.

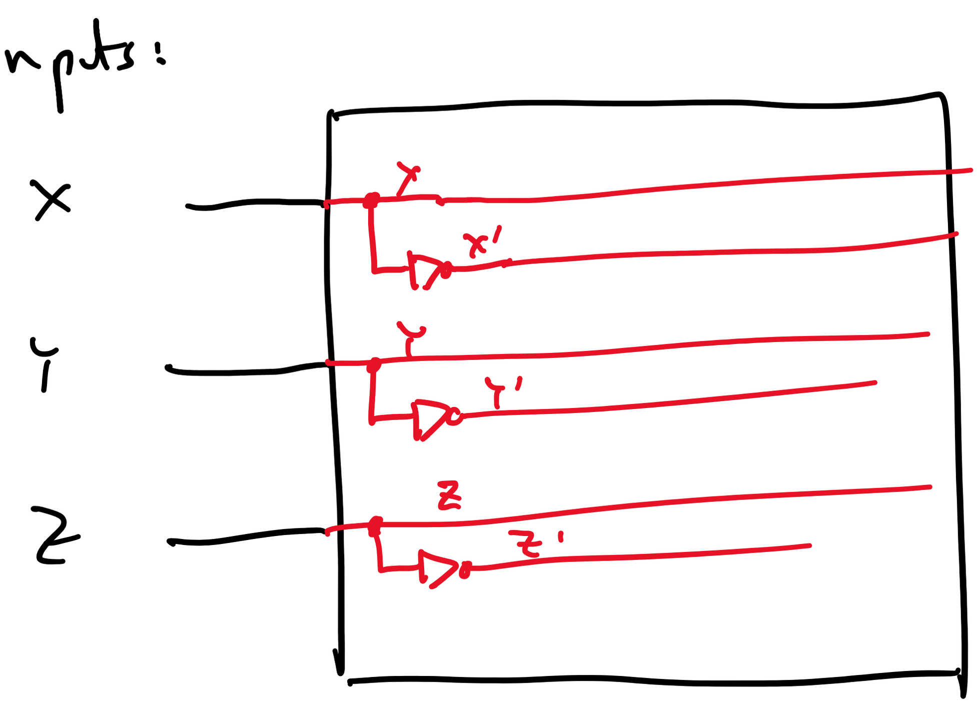
	given	dual
Uniting	$XY + XY' = X$	$(X+Y)(X+Y') = X$
Absorption	$X + XY = X$	$X(X+Y) = X$
Elimination	$X + X'Y = X+Y$	$X(X'+Y) = XY$
Consensus	$XY + X'Z + YZ = XY + X'Z$	
dual	$(X+Y)(X'+Z)(Y+Z) = (X+Y)(X'+Z)$	

These allow us to simplify expressions.

Example $XY + X'Z + YZ = XY + X'Z$

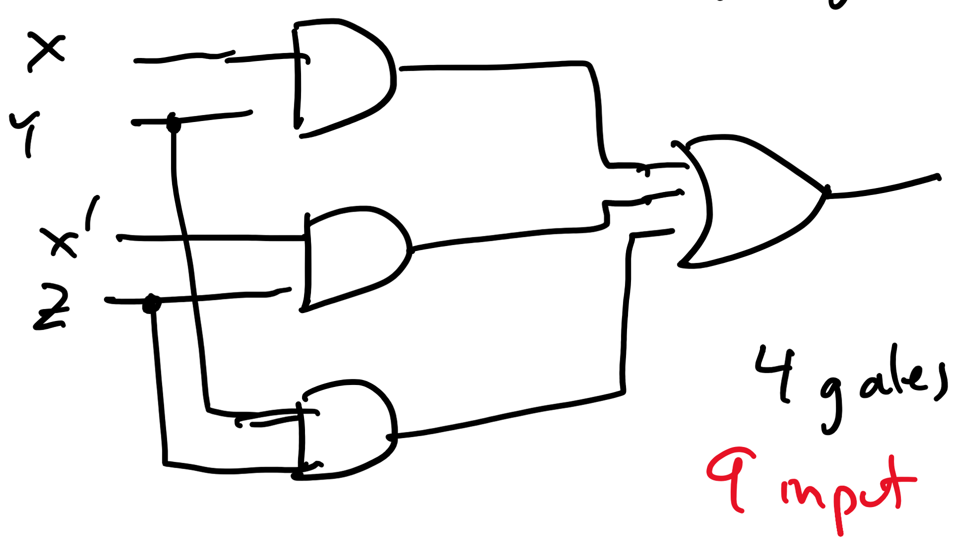
How many gates?

First, we'll assume that we always have available both the primed and unprimed versions of inputs:

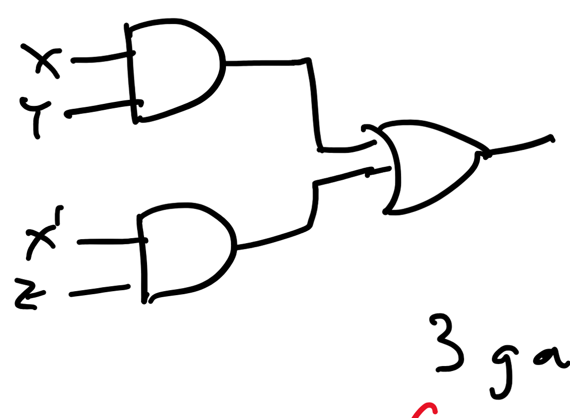


$\underbrace{XY}_{\text{AND}} + \underbrace{X'Z}_{\text{AND}} + \underbrace{YZ}_{\text{AND}}$ versus $\underbrace{XY}_{\text{AND}} + \underbrace{X'Z}_{\text{AND}}$

these OR's could be a single gate



4 gates
9 input wires



3 gates
6 input wires

Let's prove the consensus theorem for practice

Thm: $XY + X'Z + YZ = XY + X'Z$

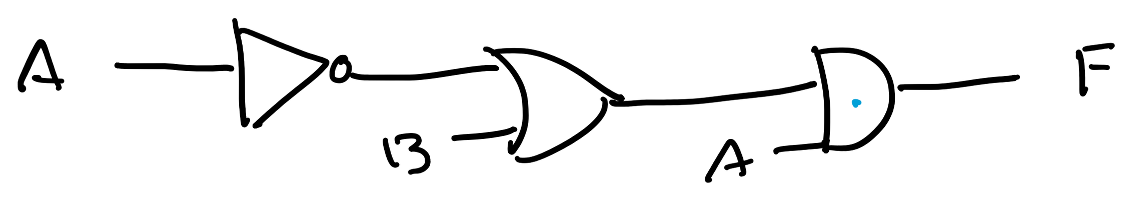
$XY + X'Z + (1)YZ$

$XY + X'Z + (X+X')YZ$

$XY + X'Z + XYZ + X'YZ$

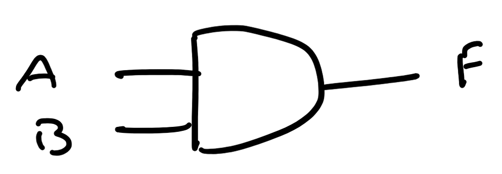
$\underbrace{XY + XYZ}_{XY} + \underbrace{X'Z + X'YZ}_{X'Z}$ commutative

Example simplification of a logic circuit



$F = (A'+B)A$ use elimination $[X(X'+Y) = XY]$

$\therefore F = AB$



saved 2 gates and some wires!

Example simplify

$Z = A'BC + A'$

Example simplify

$Z = [A + B'C + D + EF][A + B'C + (D+EF)']$

Example simplify

$Z = (AB+C)(B'D + C'E') + (AB+C)'$

hint: use elimination