

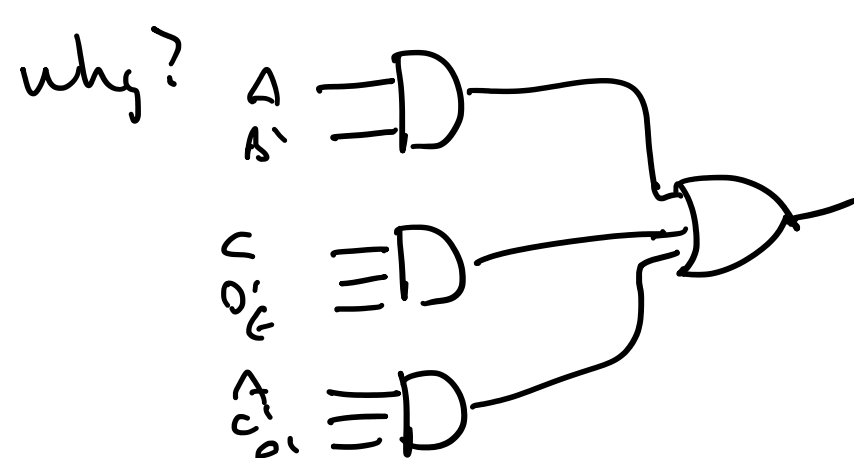
Another simplification tool

We have two standard forms we especially like

Sum-of-products (SOP)

$$AB' + CD'E + AC'E'$$

or $AB' + E$



Note $(A+B)C + DE$ is not considered sum-of-products
 $A+B$ is not a single variable multiplying other variables in a single term

To obtain sum of products, use process called "multiplying out"

Example Multiply out the following to obtain SOP

$$(A+BC)(A+D+E) \quad (\text{Is this SOP POS both neither?})$$

PRO TIP try to use 2nd distributive law first $(x+yz) = (x+y)(x+z)$

let $x=A \quad y=BC \quad z=D+E$

$$\therefore A + BC(D+E) \quad \text{2nd dist}$$

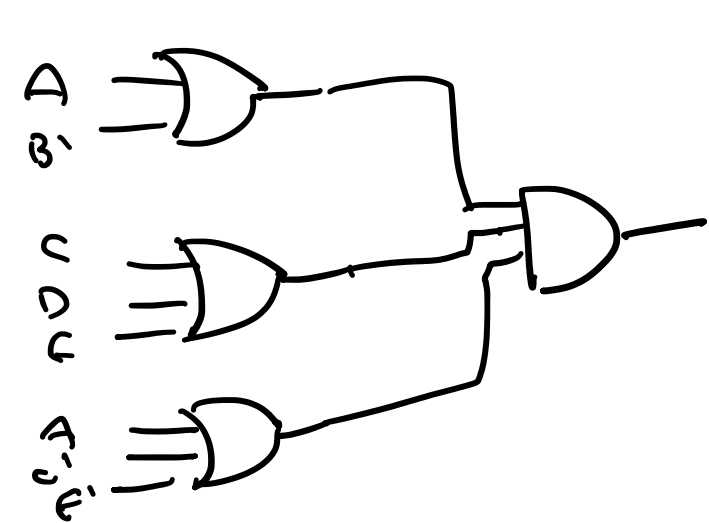
$$A + BCD + BCC \quad \text{1st dist}$$

this is SOP

Product-of-sums (POS definitely stands for product of sums)

$$(A+B')(C+D+E)(A+C'E')$$

or $(A+B)(C+D+E)F$



This is a product of sums:
 $AB'C(D'+E)$
 but this is not
 $(A+B)(C+D) + EF$
 ↑ two variables

To get POS, factor:

Thm An expression is fully factored iff it is in product-of-sums form. If not in this form, it can be factored further.

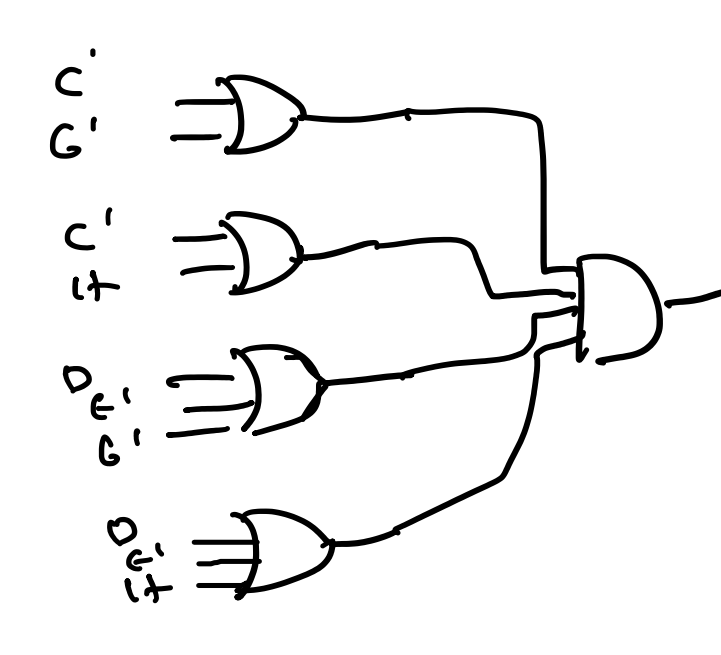
Example Factor: $A+B'CD \rightarrow$ let $A=x, B'=y, CD=z$
 then looks like $x+yz$
 Use 2nd distributive law $(x+yz) = (x+y)(x+z)$
 then $A+B'CD = (A+B')(A+CD)$

$$A+B'CD = (A+B')(A+C)(A+D) \quad \text{product of sums}$$

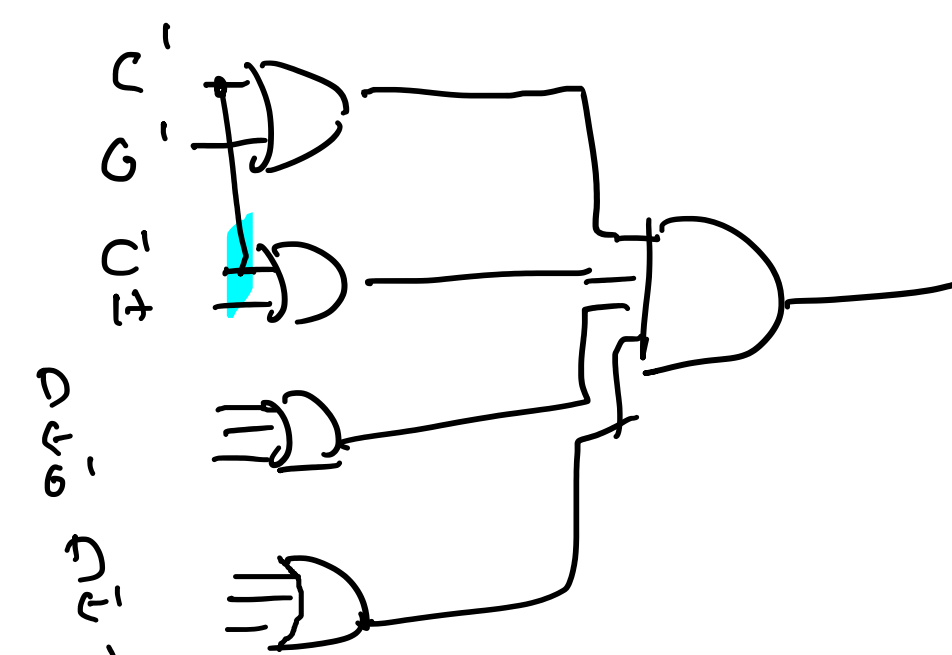
Example Factor: $A'B + C'D'$ let $x=A'B, y=C', z=D'$
 $= (x+y)(x+z)$ 2nd distr.
 $\therefore (A'B + C')(A'B + D')$ apply 2nd dist again (twice)
 to get product of sums

Example factor $C'D + C'E' + G'H$
 Use 1st distributive first:
 $C'D + C'E' + G'H = C'(D+E') + G'H$ 1st dist $(x+y+z) \times (x+z)$
 then apply 2nd distributive:
 $x+yz = (x+y)(x+z)$
 let $x=G'H, y=C', z=(D+E')$
 $(G'H + C')(G'H + (D+E'))$ repeat 2nd distr.
 $(C' + G'H)(D+E') + G'H$ commutative to make it easier to identify x,y,z
 $(C'+G'H)(C'+H)(D+E'+G')(D+E'+H)$
 Done.

looks more complicated, but can be easier (cheaper) to implement.
 POS always looks like a bunch of OR gates feeding a single AND gate



two-level ckt



these turned out to be same cost

Summary

If you are trying to obtain SOP (sum of products), multiply out (try to use 2nd dist first)

If you are trying to obtain POS (product of sums), factor (check to see if you can use first distributive first)