Conditional Probability and Independence

Conditional Probability

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- If $P(B) > 0$, we define the **conditional probability of** A **given** B to be

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Example: Rolling Dice

Example: Production Lines

A manufacturing plant produces parts on two production lines. Line 1 utilizes newer, more reliable equipment, but does not have the same production capacity as line 2. Suppose the number of functional and defective parts produced on each line are measured one day, a summarized by the following **contingency table**:

	Functional	Defective	Total
Line 1	286	14	300
Line 2	627	73	700
Total	913	87	1000

Example: Production Lines

Suppose we select a part produced from the factory on that day at random. If we let A denote the event that the part was produced on line 1, and B denote the event the part is defective, we can summarize the probabilities of the different outcomes in this experiment in a table as follows:

	B	B^c	Total
A	.014	.286	.3
A^c	.073	.627	.7
Total	.087	.913	1

What is the probability that a randomly selected part from line 1 is defective? From line 2?

Example: Production Lines

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The Multiplication Rule for Probability

The Law of Total Probability

- Recall that a family of sets $(B_i)_{i \in \mathcal{I}}$ such that $B_i \cap B_j = \emptyset$ for all $i \neq j$ and $\bigcup_{i=1}^{n} B_i = S$, then the B_i's are said to **partition** S (in this case, (B_i) is called a **partition of** S).

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- This rule generalizes to any countable partition of the sample space.

The Law of Total Probability

Probability Trees

Definition

a **probability tree** is a diagram that can be used to calculate probabilities of intersections of events that are the outcomes of experiments performed in sequence. It visualizes the process of applying the multiplication rule and the law of total probability.

Example

Recall our earlier manufacturing example, but now suppose a new part will be produced. Suppose that the probability that the next part will be produced on line 1 is .3, and that parts produced on line 1 have a 5% chance of being defective, while parts produced on line 2 have a 10% chance of being defective. draw a probability tree for this experiment.

Probability Tree Example

Bayes' Theorem

Example

We will consider a screening test for a rare disease. Suppose that the test has a **sensitivity** (probability of a positive result for a person who has the disease) of 99% and a **specificity** (probability of a negative result for a person without the disease) of 98%. Now suppose that only 1% of the population has the disease. What is the probability a randomly selected person tests positive? What is the probability a randomly selected person has the disease if the test result is positive?

Example

Independence

- Two events are called **independent** if:

- What does this imply about $P(A \mid B)$?

Independence

Informally, two events are independent if knowing that one event happened does not affect the probability that the other happens.

Example: Coin Toss

More than 2 Events: Mutual vs Pairwise Independence

- Let $A_1, A_2, \ldots, A_k$ be events. $A_1, A_2, \ldots, A_k$ are called **mutually independent** if:

  and **pairwise independent** if:

- **Pairwise independence does not imply mutual independence!**

Conditional Independence