12.1 [redacted]

A physical quantity such as mass, temperature or kinetic energy is completely determined by a single real number that specifies its magnitude.
These are called scalar quantities or simply scalars.

In contrast to this, other entities called vector quantities or vectors possess both magnitude and direction.

Vectors in the Plane.

Def. A vector is a directed line segment

Example 1 Velocity vector
Velocity of a point moving along a curved path in the plane.

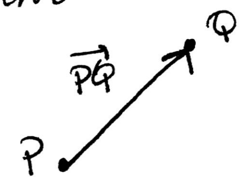
The tail of the vector is placed at the current position of the point, whose length is the speed in some agreed system of measurement, and whose direction is the direction of motion.

Example 2 Force applied to an object
Magnitude is the strength of the force and whose direction is the direction in which the force acts.

For instance, the gravitational force F exerted by the earth on a circling artificial satellite is directed toward the center of the earth and its magnitude is proportional to $1/r^2$ where r is the distance from the satellite to the center of the earth.

Notation: **v**, $\vec{v}$

If a vector extends from a point P to a point Q, we denote the vector by $\vec{PQ}$

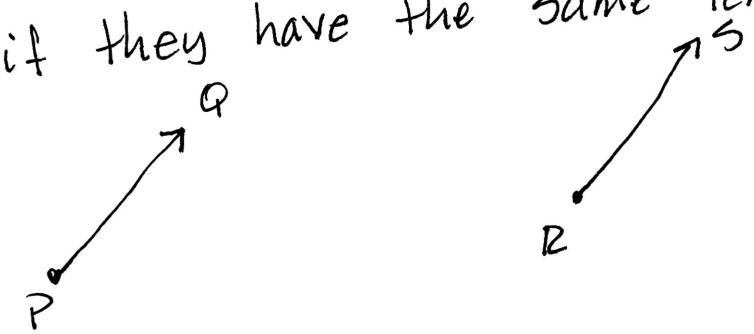


P: tail or initial point

Q: head or terminal point

$\vec{PQ}$ can be thought of as representing the displacement of a point along the line segment from P to Q.

- The <u>length</u> or magnitude of a vector $\vec{PQ}$ is denoted by the symbol $|\vec{PQ}|$
- Two vectors $\vec{PQ}$ and $\vec{RS}$ are said to be equal, $\vec{PQ} = \vec{RS}$ if they have the same length and direction.



Same vector in different positions.

- The position vector of a point P in the coordinate plane is the vector $\vec{OP}$ from the origin O to the point P.
- Any vector can be placed with its tail at the origin, and thereby becomes the position vector of the point P that lies at its head.

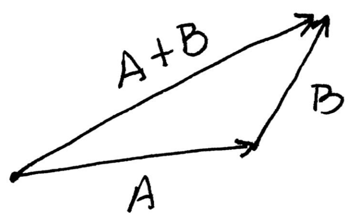
Algebraic operations on vectors.

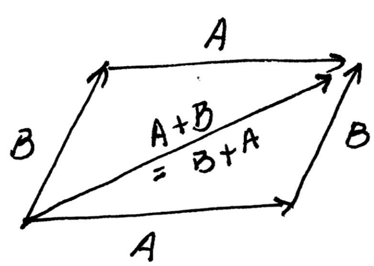
We shall discuss two algebraic operations on vectors:

- Addition of two vectors
- Scalar multiplication

First, addition:

A+B:



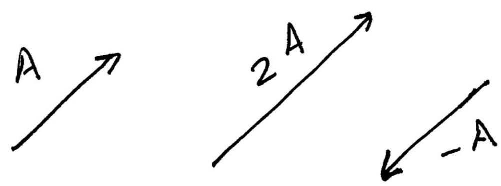


addition is commutative

$A+B = B+A$

- Now for scalar multiplication.

If $c \in \mathbb{R}$, cA is defined to be the vector which is $|c|$ times as long as A, in the same direction as A if $c>0$ and in the opposite direction if $c<0$.



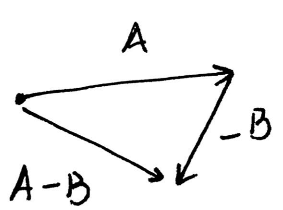
- A vector of zero length is denoted by $\vec{0}$ and called the zero vector. This vector has no direction.

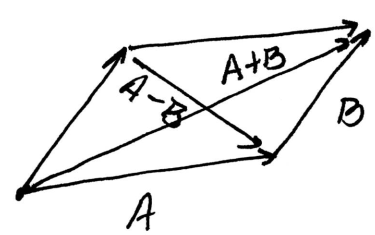
$A, 2A, -A$ are parallel vectors.

$0A = \vec{0}$ for all vectors A

$\therefore$ $\vec{0}$ is parallel to all vectors.

$(-1)B$ written $-B$ is a vector equal in length to B but having the opposite direction.





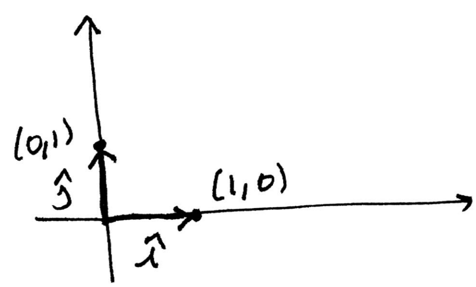
- A vector of length 1 is called a unit vector.

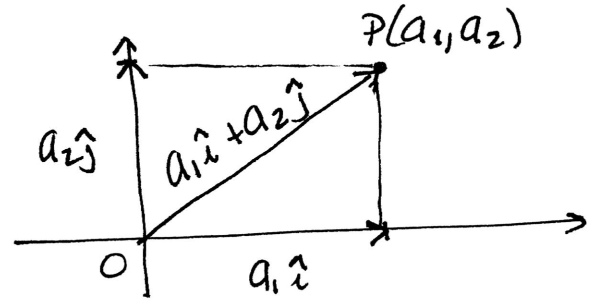
$\dfrac{A}{|A|}$ has length 1. unit vector in the same direction as A.

$-\dfrac{A}{|A|}$ unit vector in the opposite direction of A.

Unit vectors are surprisingly useful!

Working with vectors in the coordinate plane.





$\vec{OP} = a_1\hat{\imath} + a_2\hat{\jmath} = \langle a_1, a_2\rangle$

a_1: x-component of $\vec{OP}$

a_2: y-component of $\vec{OP}$

Any vector in the plane can be written as a linear combination of $\hat{\imath} = \langle 1,0\rangle$ and $\hat{\jmath} = \langle 0,1\rangle$.

$|\vec{OP}| = \sqrt{a_1^2 + a_2^2}$ magnitude of $\vec{OP}$

The value of writing any vector as a linear combination of $\hat{\imath}$ and $\hat{\jmath}$ is based on the fact that such linear combinations can be manipulated by the ordinary rules of algebra.

Thus, if $A = a_1\hat{\imath} + a_2\hat{\jmath}$ and $B = b_1\hat{\imath} + b_2\hat{\jmath}$

then
$$A+B = (a_1\hat{\imath} + a_2\hat{\jmath}) + (b_1\hat{\imath} + b_2\hat{\jmath})$$
$$= (a_1+b_1)\hat{\imath} + (a_2+b_2)\hat{\jmath}$$

Also, $cA = c(a_1\hat{\imath} + a_2\hat{\jmath}) = (ca_1)\hat{\imath} + (ca_2)\hat{\jmath}$

<u>Example</u> If $A = 3\hat{\imath} + 4\hat{\jmath}$ and $B = 2\hat{\imath} - 5\hat{\jmath}$

Find $|A|$ and express $3A - 4B$ in terms of $\hat{\imath}$ and $\hat{\jmath}$

$$|A| = \sqrt{9+16} = 5$$
$$3A - 4B = 9\hat{\imath} + 12\hat{\jmath} - 8\hat{\imath} + 20\hat{\jmath} = \hat{\imath} + 32\hat{\jmath}$$

<u>Exercise 1</u> Find a vector of length 3 which

(a) has the same direction as $5\hat{i} - 2\hat{j}$

(b) has the opposite direction to $4\hat{i} + 5\hat{j}$

(a). Let $A = 5\hat{i} - 2\hat{j}$, $|A| = \sqrt{29}$

$$u = \frac{A}{|A|} = \frac{5}{\sqrt{29}}\hat{i} - \frac{2}{\sqrt{29}}\hat{j}$$

$$\therefore B = \frac{15}{\sqrt{29}}\hat{i} - \frac{6}{\sqrt{29}}\hat{j}$$

(b). Let $A = 4\hat{i} + 5\hat{j}$, $|A| = \sqrt{16+25} = \sqrt{41}$

$$u = \frac{4}{\sqrt{41}}\hat{i} + \frac{5}{\sqrt{41}}\hat{j} \qquad -u = -\frac{4}{\sqrt{41}}\hat{i} - \frac{5}{\sqrt{41}}\hat{j}$$

$$\therefore B = \frac{-12}{\sqrt{41}}\hat{i} - \frac{15}{\sqrt{41}}\hat{j}.$$