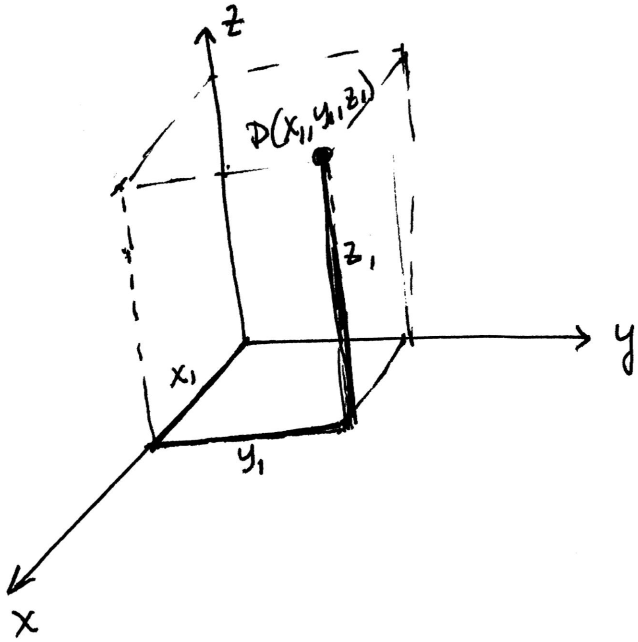
12.2

Vectors in Three Dimensional Space.



$P = (x, y, z)$

Eight octants

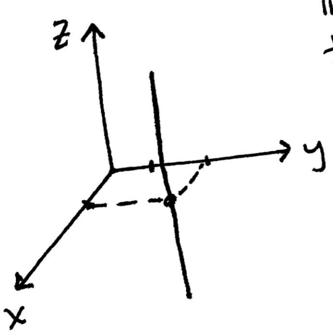
First octant: $x, y, z > 0$

$x = 0$	yz-plane
$z = 0$	xy-plane
$y = 0$	xz-plane

$x=0$, and $y=0$	:	z-axis
$y=0$ and $z=0$	:	x-axis
$x=0$ and $z=0$	:	y-axis

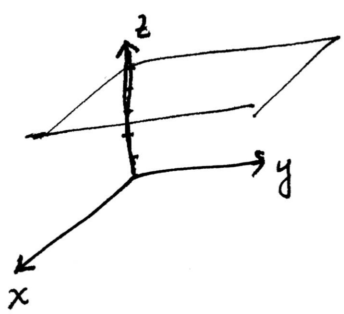
Graph: $x=1$, $y=2$

line parallel to the z-axis



Graph: $z=5$

plane parallel to the xy-plane



$\hat{i} \longleftrightarrow \langle 1,0,0 \rangle : \overrightarrow{OP}$

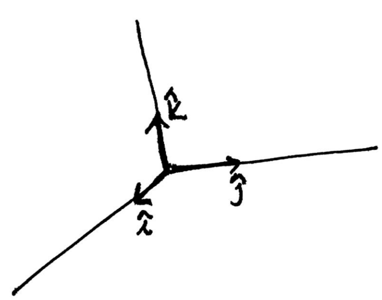
$\hat{j} \longleftrightarrow \langle 0,1,0 \rangle : \overrightarrow{OQ}$

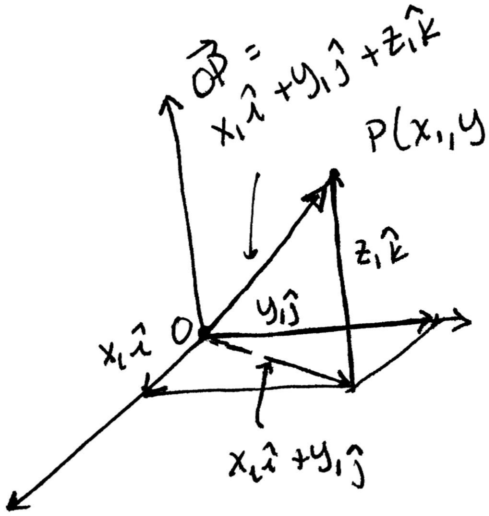
$\hat{k} \longleftrightarrow \langle 0,0,1 \rangle : \overrightarrow{OR}$

$P = (1,0,0)$

$Q = (0,1,0)$

$R = (0,0,1)$

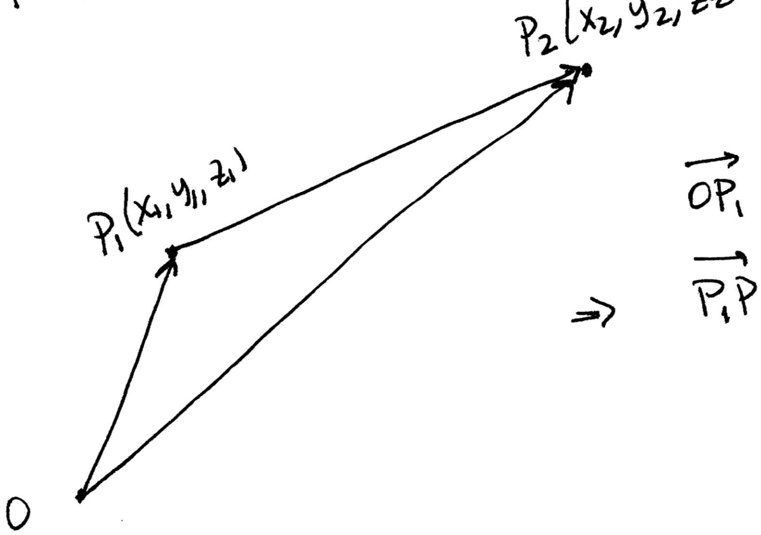




$$\vec{OP} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$|\vec{OP}| = \sqrt{x_1^2 + y_1^2 + z_1^2}$$

If $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ are any two points in $\mathbb{R}^3$, the distance between them is the length of the vector $\vec{P_1P_2}$



$$\vec{OP_1} + \vec{P_1P_2} = \vec{OP_2}$$

$$\Rightarrow \quad \vec{P_1P_2} = \vec{OP_2} - \vec{OP_1}$$

$$= (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\vec{P_1P_2} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

and therefore distance between P_1 and P_2 is

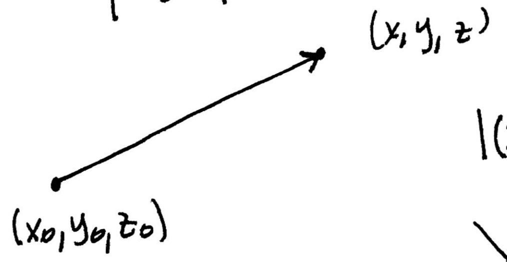
$$|\vec{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(Important distance formula)

Vectors in 3-D continues...

$P_0(x_0, y_0, z_0)$, $P(x, y, z)$

$|P_0P| = r$ sphere of radius r



$$|(x-x_0)\hat{i} + (y-y_0)\hat{j} + (z-z_0)\hat{k}| = r$$

$$\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} = r$$

- Equation of sphere centered at (x_0, y_0, z_0) of radius r is:

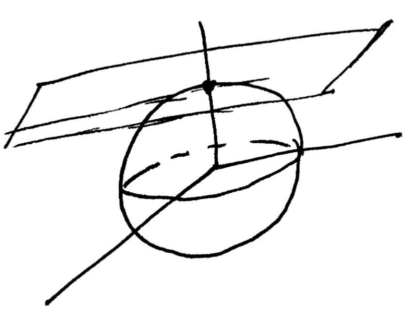
$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

- Equation of the ball centered at (x_0, y_0, z_0) of radius r is:

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 \le r^2$$

<u>Example</u> Give a geometric description of the points that lie on the intersection of the sphere $x^2+y^2+z^2=36$ and the plane (a) $z=6$ (b) $z=3$

(a)



$x^2+y^2=0$

$\Rightarrow x=0,\ y=0$

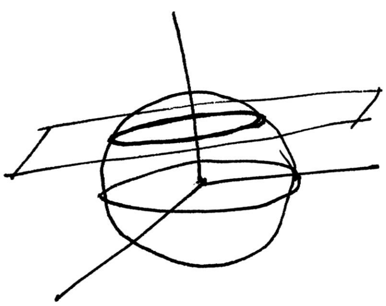
$z=6$ intersection is the point $(0,0,6)$

(b) $x^2+y^2+z^2=36$

$z=3$

$x^2+y^2+9=36 \quad \Rightarrow \quad x^2+y^2=27$

circle centered at $(0,0,3)$, radius $\sqrt{27}$
it lies in the horizontal plane $z=3$.



<u>Example</u> Let $P=(1,5,0)$, $Q(3,11,2)$. Find two unit vectors parallel to $\overrightarrow{PQ}$

$\overrightarrow{PQ} = \langle 2,6,2\rangle$

$|\overrightarrow{PQ}| = \sqrt{4+36+4} = 2\sqrt{11}$

$\pm u = \pm \dfrac{\langle 2,6,2\rangle}{2\sqrt{11}} = \pm \left\langle \dfrac{1}{\sqrt{11}}, \dfrac{3}{\sqrt{11}}, \dfrac{1}{\sqrt{11}} \right\rangle$

or $\left\langle \dfrac{1}{\sqrt{11}}, \dfrac{3}{\sqrt{11}}, \dfrac{1}{\sqrt{11}} \right\rangle$, $\left\langle -\dfrac{1}{\sqrt{11}}, \dfrac{-3}{\sqrt{11}}, -\dfrac{1}{\sqrt{11}} \right\rangle$