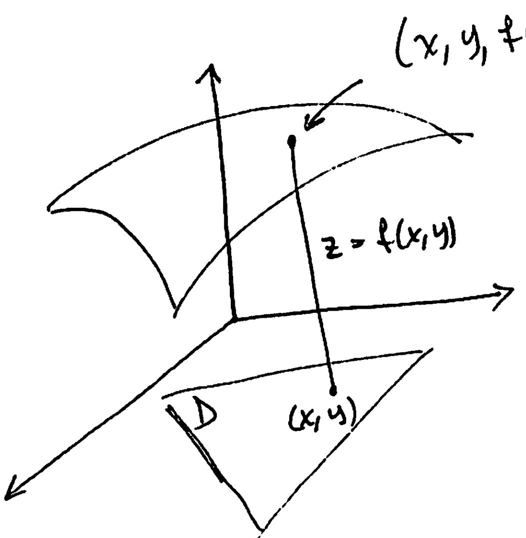
13.2 Graphs and Level Curves

Def A function $z = f(x,y)$ assigns to each point (x,y) in a set D in $\mathbb{R}^2$ a unique real number z in a subset of $\mathbb{R}$.

The set D is the domain of f

The range of f is the set of real numbers z that are assumed as the points (x,y) vary over the domain.



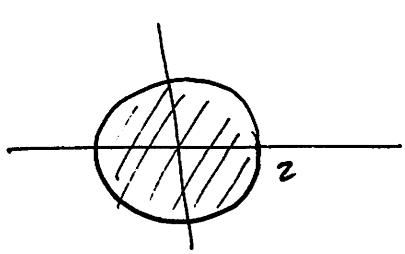
Ex. Find the domain of the function

$g(x,y) = \sqrt{4-x^2-y^2}$

(x,y) is in $D \iff 4-x^2-y^2 \geq 0 \iff x^2+y^2 \leq 4$

$\therefore \quad D = \{(x,y) \mid x^2+y^2 \leq 4\}$

$D =$



disk of radius 2.

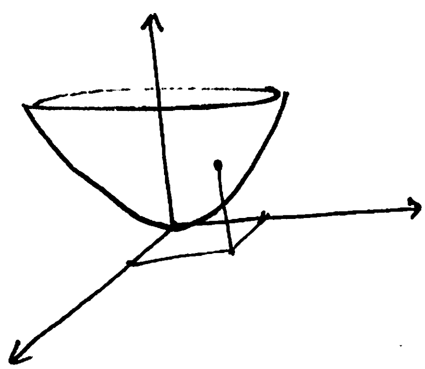
Def Graph of f:

Graph of f = $\{(x,y,z) \mid z = f(x,y)\}$

Ex. Graph the function

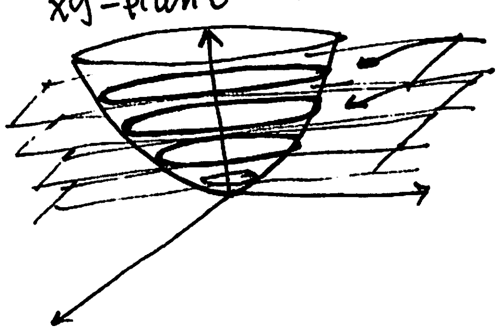
$$g(x,y) = x^2 + y^2$$

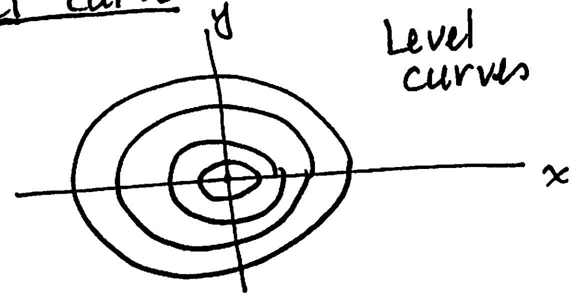
$$z = x^2 + y^2 \qquad \text{(paraboloid).}$$



Level Curves Let $z = f(x,y)$ be a function, and $z = z_0$ be a plane.

The intersection of $z = f(x,y)$ and $z = z_0$ is a curve along the graph of f at height z_0, when this contour curve is projected onto the xy-plane, the result is the curve $f(x,y) = z_0$. This curve in the xy-plane is called a level curve.





Ex Represent the given function by drawing a few level curves, and try to visualize the surface from the resulting level curves.

$$f(x,y) = y - x^2$$

Lets graph a few level curves:

$z = -2$	$y - x^2 = -2$	$\Rightarrow \quad y = x^2 - 2$
$z = -1$	$y - x^2 = -1$	$\Rightarrow \quad y = x^2 - 1$
$z = 0$	$y - x^2 = 0$	$\Rightarrow \quad y = x^2$
$z = 1$	$y - x^2 = 1$	$\Rightarrow \quad y = x^2 + 1$
$z = 2$	$y - x^2 = 2$	$\Rightarrow \quad y = x^2 + 2$