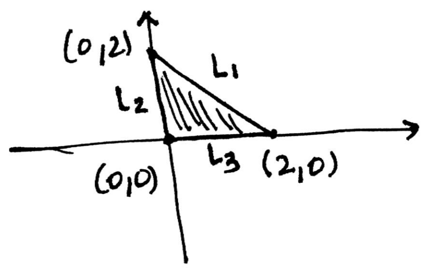
13.8 Cont... Absolute Maximum and Minimum Values

Def. Let f be defined on a set R in $\mathbb{R}^2$ containing the point (a,b). If $f(a,b) \geq f(x,y)$ $\forall$ (x,y) in R then $f(a,b)$ is an absolute maximum value of f in R.
If $f(a,b) \leq f(x,y)$ $\forall$ (x,y) in R, then $f(a,b)$ is an absolute minimum value of f in R.

Ex. $f(x,y) = x^2 + y^2 - 2x - 2y$; R is the closed region bounded by the triangle with vertices $(0,0)$, $(2,0)$, $(0,2)$.

R:



Line between the points $(0,2)$ and $(2,0)$ is:

$y = -x + 2$ $\quad 0 \leq x \leq 2$, $\quad 0 \leq y \leq 2$

Line between $(0,2)$ and $(0,0)$

$x = 0 \quad 0 \leq y \leq 2$

Line between $(0,0)$ and $(2,0)$

$y = 0 \quad 0 \leq x \leq 2$

First we look for critical points in the interior of R:

$f_x = 2x - 2$

$f_y = 2y - 2$

setting the partial derivatives equal to zero we get:

$2x - 2 = 0$
$2y - 2 = 0$ $\Rightarrow$ $x = 1$, $y = 1$

$(1,1)$ is inside the region R
actually is on the boundary

Now, we look at the boundary of the region.

L_1: $y = -x + 2$ $\quad 0 \leq x \leq 2, \quad 0 \leq y \leq 2$

$$f(x,y) = x^2 + y^2 - 2x - 2y$$
$$= x^2 + (-x+2)^2 - 2x - 2(-x+2)$$
$$= x^2 + x^2 + 4 - 4x - 2x + 2x - 4$$
$$= 2x^2 - 4x$$

candidates for abs max or abs min	The function evaluated at the candidates	
• $(1,1)$	$f(1,1) = -2$	←
• $(0,2)$	$f(0,2) = 0$	←
• $(2,0)$	$f(2,0) = 0$	←
• $(0,1)$	$f(0,1) = -1$	
• $(0,0)$	$f(0,0) = 0$	←
• $(1,0)$	$f(1,0) = -1$	

Table ⊛

$\therefore\ f(x,y) = g(x) = 2x^2-4x \qquad 0 \le x \le 2$

$g'(x) = 4x-4 \overset{\text{set}}{=} 0$

$x = 1 \Rightarrow \qquad (1,1)$

$y = -x+2 = -\frac{1}{\cancel{2}}+2 = 1$

at the endpoints we obtain: $\quad (0,2)$ $\quad (2,0)$

consider now L_2:

$x=0 \qquad 0 \le y \le 2$

$f(x,y) = x^2+y^2-2x-2y$
$= y^2-2y \qquad \Rightarrow (0,1)$

$f(x,y) = h(y) = y^2-2y$

$h'(y) = 2y-2$

$2y-2=0 \Rightarrow y=1$

at the endpoints we obtain $\quad (0,0)$ $\quad (0,2)$

L_3: $\quad y=0 \qquad 0 \le x \le 2$

$f(x,y) = x^2+y^2-2x-2y$
$= x^2-2x \qquad \Rightarrow (1,0)$

$f(x,y) = q(x) = x^2-2x$

$q'(x) = 2x-2$

$2x-2=0 \Rightarrow x=1$

at the endpoints we obtain $\quad (0,0)$ $\quad (2,0)$

Looking at the results on Table (*) we obtain: The absolute maximum of f on R is equal to $\underline{0}$ and it occurs at the points: $(0,2)$, $(2,0)$ and $(0,0)$. The absolute minimum is $\underline{-2}$ and it occurs at $(1,1)$