15.1 Level curves & graphs $\mathbb{R}^2 \to \mathbb{R}$

Def: A function $z = f(x,y)$ assigns to each point (x,y) in a set D in $\mathbb{R}^2$ a unique real number z in a subset of $\mathbb{R}$. The set D is the domain of f. The range of f is the set of Real numbers z that are assumed as the points (x,y) vary over the domain.

Def: The graph of $f = \{(x,y,z) \mid z = f(x,y)\}$

These graphs can be represented by surfaces in $\mathbb{R}^3$. However, these surfaces are often difficult to picture. We use Level curves to help us picture these sufaces.

Ex: What does the graph of $F(x,y) = \underline{x^2+y^2}$ look like?

Let us find the level curves at

$z=0$, $z=1$, $z=4$, $z=9$.

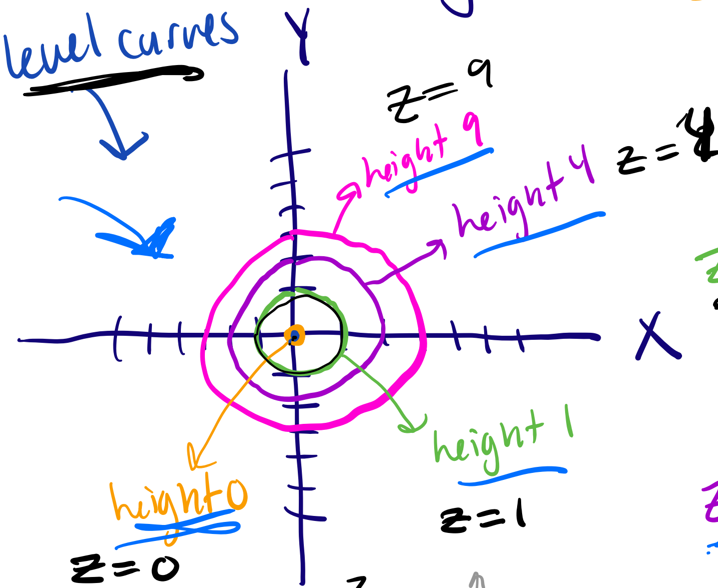
$z=0$	$z=1$	$z=4$	$z=9$
all the x,y values that correspond to a "height" of 0	all the x,y values that correspond to a "height" of 1	all the x,y values that correspond to a "height" of 4	all the x,y values that correspond to a "height" of 9

* all points (x,y) where $f(x,y)=0$

To Find the level curve at $z=0$, we plug 0 in for $f(x,y)$ and see what x,y coordinates will correspond to that height.

$f(x,y) = x^2+y^2$

level curves



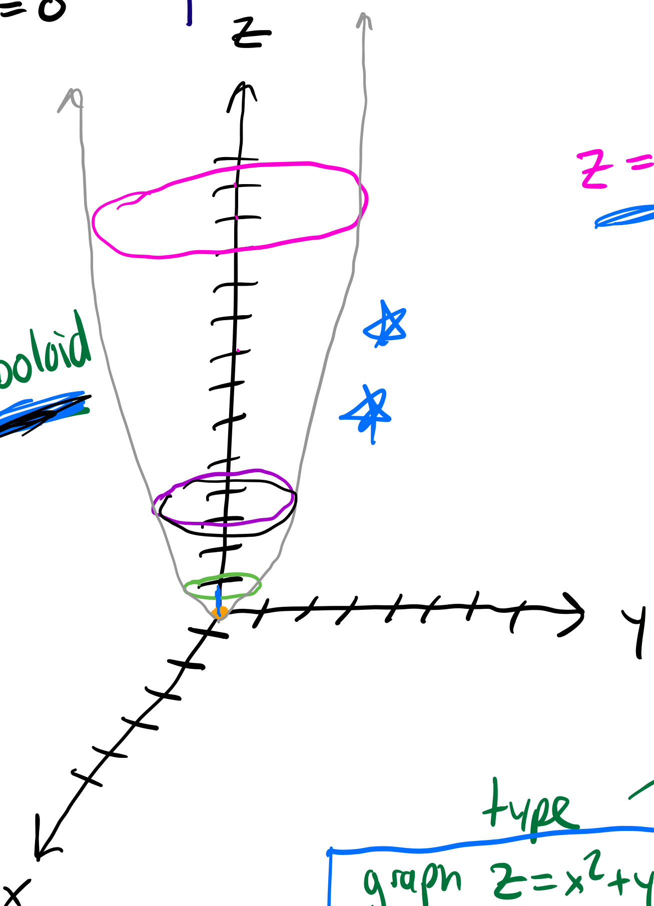
$z=0$: $0 = x^2+y^2$ only $x=0,\ y=0$

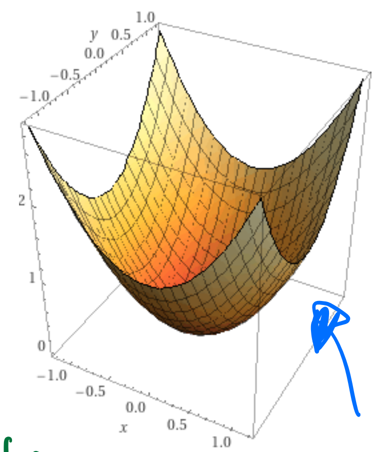
$z=1$: $\boxed{1 = x^2+y^2}$ circle of radius 1

$z=4$: $4 = x^2+y^2$ circle of radius 2

$z=9$: $9 = x^2+y^2$ circle of radius 3

Paraboloid





type $\boxed{\text{graph } z=x^2+y^2}$ into wolfram alpha

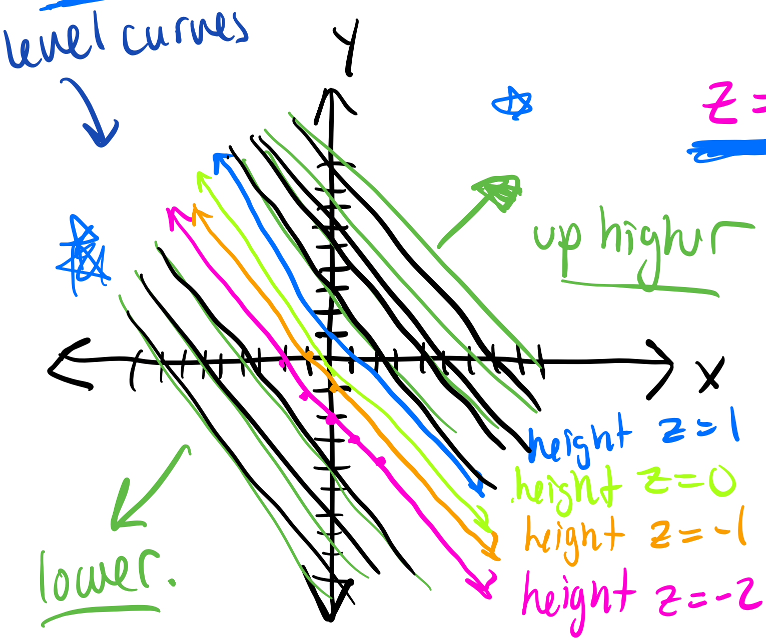
Ex: What does $f(x,y) = x+y$

look like?

Let's find the level curves for heights

$z=-2$, $z=-1$, $z=0$, $z=1$

$f(x,y) = x+y$



$z=-2$

$-2 = x+y$

↳ $y = -x-2$

slope $m=-1$

yint $=-2$

$z=-1$

$-1 = x+y$

$y = -x-1$

slope $m=-1$

yint $=-1$

$z=0$

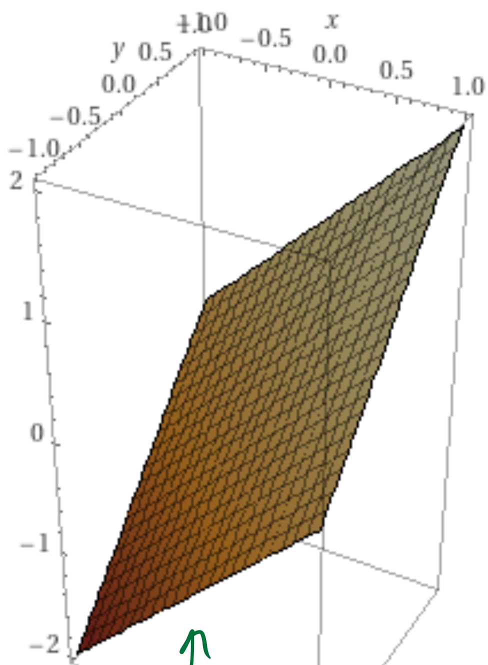
$0 = x+y$

$y = -x$

$z=1$

$1 = x+y$

$y = -x+1$



(graph $z = x+y$)

↳ type into wolfram alpha

* pg. 928
problem 34.

↳ graphs ↳ level curves

One more example of level curves:

$$f(x,y) = z = \sqrt{4 - x^2 - y^2}$$

$4 - x^2 - y^2 \geq 0$

$4 \geq x^2 + y^2$

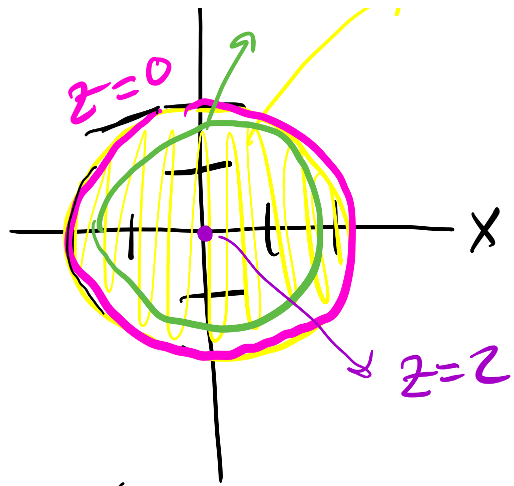
* note in class we said the domain $D := x^2 + y^2 \leq 4$ — disc of radius 2

Let's find level curves at $z = 0$, $z = 1$, $z = 2$

Y

$z=1$ → Domain

$z=0$



square both sides

$$0 = \sqrt{4-x^2-y^2}$$

$$0 = 4-x^2-y^2$$

$$x^2+y^2=4$$

circle of radius 2

$z=1$

$$1 = \sqrt{4-x^2-y^2}$$

$$1 = 4-x^2-y^2$$

$$x^2+y^2=3$$

circle of $r=\sqrt{3}$

$\sqrt{3} \approx 1.73$

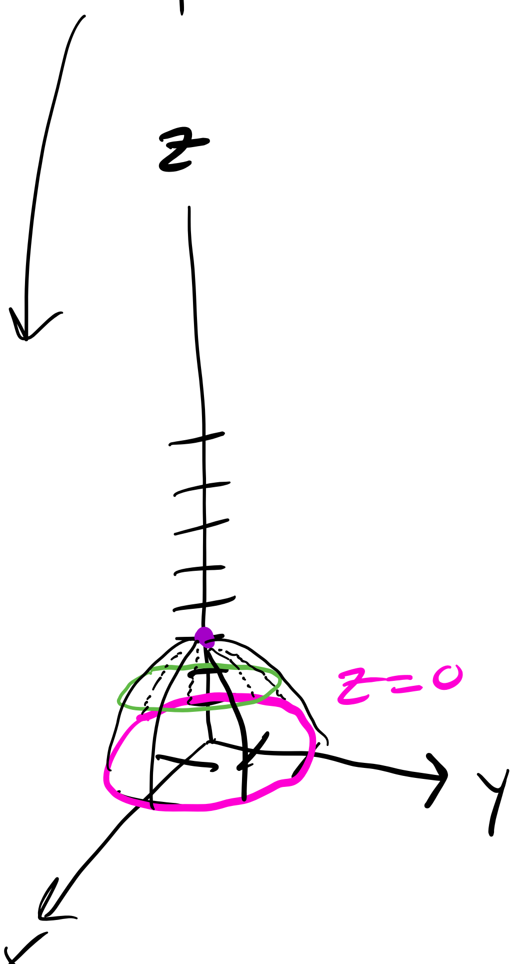
$z=2$

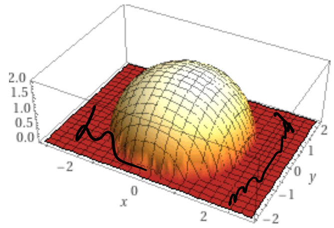
$$2 = \sqrt{4-x^2-y^2}$$

$$4 = 4-x^2-y^2$$

$$x^2+y^2=0$$

$(0,0)$





graph $z=\sqrt{4-x^2-y^2}$

wolfram alpha

rudman.25@osu.edu.